Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/62**

Paper 6 **October/November 2018**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

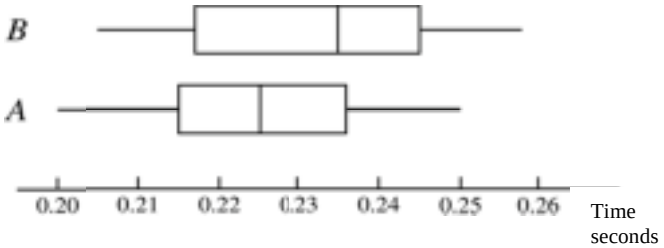
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	$\frac{11!}{4!4!2!}$	**M1**	$\frac{11!}{4!\times k}$ *or* $\frac{11!}{2!\times k}$, k a positive integer
	= 34650	**A1**	Correct final answer
		2	
1(ii)	**Method 1**		
	P(SS) = $\frac{4}{11}\times\frac{3}{10}=\frac{12}{110}$ (= 0.10911)	**B1**	One of P(SS), P(PP) or P(II) correct, allow unsimplified
	P(PP) = $\frac{2}{11}\times\frac{1}{10}=\frac{2}{110}$ (= 0.01818) P(II) = $\frac{4}{11}\times\frac{3}{10}=\frac{12}{110}$ (= 0.10911) $\frac{4}{11}\times\frac{3}{10}$	**M1**	Sum of probabilities from 3 appropriate identifiable scenarios (either by labelling or of form $\frac{4}{11}\times\frac{a}{b}+\frac{2}{11}\times\frac{c}{b}+\frac{4}{11}\times\frac{a}{b}$ where a = 4 *or* 3, b = 11 *or* 10, c = 2 *or* 1)
	Total = $\frac{26}{110}=\frac{13}{55}$ oe (0.236)	**A1**	Correct final answer
	Method 2		
	Total number of selections = $^{11}C_2$ = 55 Selections with 2 Ps = 1	**B1**	Seen as the denominator of fraction (no extra terms) allow unsimplified
	Selections with 2 Ss = $^4C_2$ = 6 Selections with 2 Is = $^4C_2$ = 6,	**M1**	Sum of 3 appropriate identifiable scenarios (either by labelling or values, condone use of permutations. May be implied by 2,12,12)
	Total selections with 2 letters the same = 13 Probability of 2 letters the same = $\frac{13}{55}$ oe (0.236)	**A1**	Correct final answer, without use of permutations
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">2(i)</td><td>median = 0.225;
LQ = 0.215: UQ = 0.236</td><td>B1</td><td>Correct median (Q_2)</td></tr>
<tr><td>IQR = 0.236 – 0.215</td><td>M1</td><td>0.232 < UQ (Q_3) < 0.238 – 0.204 < LQ (Q_1) < 0.219</td></tr>
<tr><td>= 0.021</td><td>A1</td><td>www
Omission of all decimal points MR-1
<u>If M0 awarded</u>
SCB1 for both LQ = 0.215: UQ = 0.236 seen</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="4">2(ii)</td><td> 
</td><td>B1</td><td>Linear scale between 0.20 to 0.26 (condone omission of 0.26) axis labelled (time and seconds), at least one box plot attempted, no lines through boxes, whiskers not at corner of boxes</td></tr>
<tr><td></td><td>B1 ft</td><td>Labelled correct graph for A, (ft their median/quartiles), condone lines through boxes, whiskers at corner of boxes</td></tr>
<tr><td><table>
<tr><td>A</td><td>0.200</td><td>0.215</td><td>0.225</td><td>0.236</td><td>0.250</td></tr>
<tr><td>B</td><td>0.205</td><td>0.217</td><td>0.235</td><td>0.245</td><td>0.258</td></tr>
</table></td><td>B1</td><td>Labelled correct graph for B, condone lines through boxes, whiskers at corner of boxes

SC If B0B0 scored because graphs not labelled/labels reversed SCB1 if both ‘correct’

Penalty MR-1 if graphs plotted on separate axes unless both scales align exactly.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
3(i)	**Method 1**		
	$P(3) + P(4) + P(5) = {}^5C_3\ 0.75^3 \times 0.25^2\ +$	**M1**	One binomial term ${}^5C_x p^x(1-p)^{5-x}$, $x \neq 0$ or 5, any p
	${}^5C_4\ 0.75^4 \times 0.25^1\ + {}^5C_5\ 0.75^5 \times 0.25^0$	**M1**	Correct unsimplified expression
	$= 0.26367 + 0.39551 + 0.23730$ $= 0.896$ (459/512)	**A1**	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
	Method 2		
	$1 - P(0) - P(1) - P(2) = 1 - {}^5C_0\ 0.75^0 \times 0.25^5$	**M1**	One binomial term ${}^5C_x p^x(1-p)^{5-x}$, $x \neq 0$ or 5, any p
	$- {}^5C_1\ 0.75^1 \times 0.25^4 - {}^5C_2\ 0.75^2 \times 0.25^3$	**M1**	Correct simplified expression
	$= 1 - 0.00097656 - 0.014648 - 0.087891$ $= 0.896$ (459/512)	**A1**	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
		3	

Question	Answer	Marks	Guidance
3(ii)	**Method 1**		
	P(C,C) + P(C,C′) + P(C′,C) 0.8 × 0.9	**B1**	Unsimplified prob completed on both days
	0.8 × 0.1 + 0.2 × 0.6	**M1**	Unsimplified prob 0.8 × *a* + 0.2 × *b*, *a* = 0.1or 0.4, *b* = 0.6 or 0.9
	= 0.92 oe	**A1**	Correct final answer
	Method 2		
	1 – P(C′,C′) = 1 – 0.2 × 0.4	**B1**	Unsimplified prob completed on no days
		M1	1 – 0.2 × *a*, *a*=0.1or 0.4 allow unsimplified
	= 0.92	**A1**	Correct final answer
		3	

Question	Answer	Marks	Guidance
4(i)	5! × 6! ×2	**B1**	k×5! or *m*×6! (*k*,*m* integer, *k*,*m* ⩾ 1), no inappropriate addition
		B1	*n* × 5! × 6! (*n* integer, *n* ⩾ 1), no inappropriate addition
	= 172800	**B1**	Correct final answer, isw rounding (www scores B3) All marks based on their **final** answer
		3	

Question	Answer	Marks	Guidance
4(ii)	… G … G … G … G … G … G … No. ways girls placed × No. ways boys placed in gaps =	**M1**	$k \times 6!$ or $k \times {}^7P_5$ (k is an integer, $k \geqslant 1$) no inappropriate add. (${}^7P_5 \equiv 7 \times 6 \times 5 \times 4 \times 3$ or ${}^7C_5 \times 5!$)
	$6! \times {}^7P_5$	**M1**	Correct unsimplified expression
	= 1814400	**A1**	Correct exact final answer (ignore subsequent rounding)
		3	

Question	Answer	Marks	Guidance
5(i)	$\frac{15.5 \times 12 + 910}{12 + 20}$	**M1**	Unsimplified total age divided by *their* total members (not 12, 20 or 2)
	=34.25 or 34¼ (years)	**A1**	Correct exact answer (isw rounding), oe (34 years 3 months)
		2	
5(ii)	Considering Juniors: variance = $\frac{\sum x^2}{12} - 15.5^2 = 1.2^2$	**M1**	$\frac{\sum x^2}{k} - 15.5^2 = 1.2^2$, k = 12 or 20
	$\sum x^2 = 2900.28$	**A1**	Answer wrt 2900
	Considering whole group: $\sum z^2 = \sum x^2 + \sum y^2 = 2900.28 + 42850 = 45750$ Variance = $\frac{\sum z^2}{32} - \mu^2 = \frac{their\ 45750}{12 + 20} - (their\ 34.25)^2$ (= 256.63)	**M1**	*Their* 45750 > 42850 (not 85700 or rounding to 1.8×10^9) in correct variance or std deviation formula (Σx^2 and addition may not be seen)
	s d = 16.0(2)	**A1**	Correct final answer, condone 16.03
		4	

Question	Answer	Marks	Guidance
6(i)	x: –2, –1, 0, 1, 2, 3 p: $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{3}{12}$, $\frac{2}{12}$, $\frac{1}{12}$	**B1**	–2, –1, 0, 1, 2, 3 seen as top line of a pdf table with at least 1 probability OR attempting to evaluate P(–2), P(–1), P(0), P(1), P(2), P(3) (condone additional values with p=0 stated)
		B1	At least 4 probs correct (need not be in table)
		B1	All probs correct in a table
		3	
6(ii)	$\text{E}(X) = \frac{-2\times1-1\times2+0+1\times3+2\times2+1\times3}{12} = 0.5$	**M1**	Unsimplified expression for mean using *their* pdf table (or correct) with at least 2 non-zero values (may be seen in variance). Numerator terms may be implied by values.
	$\text{Var}(X) = \frac{(-2)^2\times1+(-1)^2\times2+1^2\times3+2^2\times2+3^2\times1}{12} - (\textit{their}\ 0.5)^2$	**M1**	Unsimplified expression for variance using *their* pdf table (or correct) with at least 2 non-zero values and *their* E(X). Numerator terms may be implied by values. If $-k^2$ is seen for $(-k)^2$, the method must be confirmed by seeing value used correctly
	26/12 −1/4 = 23/12	**A1**	Correct final answer
		3	

Question	Answer	Marks	Guidance
6(iii)	**Method 1**		
	P(X non-zero) = 9/12	**B1ft**	If Binomial distribution used 0/3 P(X non-zero) ft from *their* pdf table, Σp=1 oe
	P($X = 1 \mid X$ non-zero) = $\dfrac{\mathrm{P}(X = 1 \cap X \text{ non-zero})}{\mathrm{P}(X \text{ non-zero})} = \dfrac{3/12}{9/12}$	**M1**	*Their* P(X = 1)/*their* P(X non-zero) from *their* pdf table oe
	= 1/3 oe	**A1**	Correct final answer www
	Method 2		
	P($X = 1 \mid X$ non-zero) = $\dfrac{\text{Number of outcomes} = 1}{\text{Number of non-zero outcomes}}$	**B1ft**	Number of non-zero outcomes (expect 9) ft from *their* outcome table or pdf table numerators oe
		M1	*a/b*, *a* = *their* 3 from *their* outcome table or pdf table numerators, *b* = *their* 9 (not 12)
	$= \dfrac{3}{9} = \dfrac{1}{3}$ oe	**A1**	Correct final answer www
		3	

Question	Answer	Marks	Guidance
7(a)(i)	$P(X < 4) = P\left(Z < \frac{4-3.24}{0.96}\right)$	**M1**	±Standardisation formula, no cc, no sq rt, no square
	$= P\left(Z < 0.7917\right) = 0.7858$	**A1**	$0.7855 < p \leqslant 0.7858$ or p = 0.786 Cao (implies M1A1 awarded), may be seen used in calculation
	their 0.7858 × 365 = 286 (or 287)	**B1ft**	*Their* probability × 365 provided 4sf probability **seen**. FT answer rounded or truncated to nearest integer. No approximation notation used.
		3	
7(a)(ii)	$P(X < k) = P(Z < \frac{k-3.24}{0.96}) = 0.8$	**B1**	(z=) ± 0.842 seen
	$\frac{k-3.24}{0.96} = 0.842$	**M1**	$z = \pm\frac{k-3.24}{0.96}$, allow cc, sq rt or square equated to a *z*-value (0.7881, 0.2119, 0.158, 0.8, 0.2 etc. are not acceptable)
	$k = 4.05$	**A1**	Correct final answer, www
		3	
7(a)(iii)	$P(-1.5 < Z < 1.5) =$	**M1**	$\Phi(z = 1.5)$ or $\Phi(z = -1.5)$ seen used or $p = 0.9332$ seen
	$\Phi(1.5) - \Phi(-1.5) = 2\Phi(1.5) - 1$ = 2 × 0.9332 – 1 oe	**M1**	Correct final area expression using *their* probabilities
	= 0.866	**A1**	Correct final answer
		3	

Question	Answer	Marks	Guidance
7(b)	$P(Y > 0) = P\left(Z > \frac{0-\mu}{\sigma}\right) \equiv P\left(Z > \frac{0-\mu}{3\mu/4}\right)$ or $P\left(Z > \frac{0-\left(\frac{4\sigma}{3}\right)}{\sigma}\right)$	**M1**	±Standardisation attempt in terms of one variable no sq rt or square, condone ±0.5 as cc
	$= P(Z > -4/3)$	**A1**	Correct unsimplified standardisation, no variables
	$= 0.909$	**A1**	Correct final answer
		3	

Alternative methods for Question 1(ii)

Method 3

$P(S,S') = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$

$P(P,P') = \frac{2}{11} \times \frac{9}{10} = \frac{18}{110}$

$P(I,I') = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$

$P(M,M') = \frac{1}{11} \times \frac{10}{10} = \frac{10}{110}$

$Total = \frac{84}{110}$

$P(Same) = 1 - \frac{84}{110} = \frac{26}{110}$

B1 one of products correct
M1 1 – sum of probabilities from 4 appropriate scenarios
A1 Correct final answer

Method 4

PP' = $\frac{2\times9}{2} = 9$

SS' = $\frac{4\times7}{2} = 14$

II' = $\frac{4\times7}{2} = 14$

MM' = $\frac{1\times10}{2} = 5$

Total number of ways = $\frac{10\times11}{2} = 55$

Number of ways of letters repeating = 55 – (9 + 14 + 14 + 5) = 13

P(Same) = $\frac{13}{55}$

B1 $^{11}C_2$ seen as the denominator of fraction (no extra terms) allow unsimplified
M1 1 – sum of 4 appropriate scenarios
A1 Correct final answer